Examination Cover Sheet

Princeton University Undergraduate Honor Committee

January 17, 2007

Course Name & Number: PHY105
Professor: Lyman Page

Date: January 22, 2007
Time: 1:30 PM

This examination is administered under the Princeton University Honor Code. Students should sit one seat apart from each other, if possible, and refrain from talking to other students during the exam. All suspected violations of the Honor Code must be reported to the Honor Committee Chair at honor@princeton.edu.

The checked items are permitted for use on this examination. Any item that is not checked may not be used and should not be in your working space. Assume items not on this list are not allowed for use on this examination. Please place items you will not need out of view in your bag or under your working space at this time. University policy does not allow the use of electronic devices such as cell phones, PDAs, laptops, MP3 players, iPods, etc. during examinations. Students may not wear headphones during an examination.

- ☐ Course textbooks: No
- ☐ Course Notes: No
- ☐ Other books/printed materials: No
- ☒ Formula Sheet: Yes, but only the one provided
- ☐ Comments on use of printed aids: None

Students may only leave the examination room for a very brief period without the explicit permission of the instructor. The exam may not be taken outside of the examination room.

This exam is a timed examination. You will have

3 hours 0 minutes

to complete this exam.

During the examination, the Professor or a preceptor will be available at the following location:

On the cover of your first booklet, write and sign the Honor Code pledge: "I pledge my honor that I have not violated the Honor Code during this examination"

This exam consists of **seven** problems. When we begin, check to see that this copy of the exam has all seven. Use the same exam booklet for all problems, continuing to another booklet if necessary. **Print** your name on **each** booklet as you start it.

USE OF CALCULATORS IS NOT PERMITTED!!

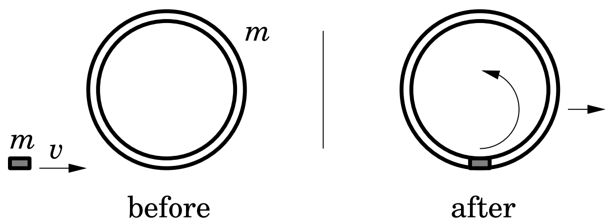
At the end of the exam, indicate clearly on the cover of your first exam booklet how many booklets you used.

Some useful test-taking hints:

- You may not be able to complete every problem. Keep moving – do what you know first.
- Make your answer clear by circling it.
- Use symbols rather than numbers wherever possible and CHECK UNITS.
- Whenever possible, check whether an answer or intermediate result makes sense before moving on.
- There is a list of formulas on a page you can tear out attached to the exam. Use it as a reminder of details. Don't try to do problems by searching through the sheet!
- If you get stuck on an early part of a problem, check the later parts — some may be independent and doable.
- If you get stuck on an early part of a problem, and a later part depends on it, **clearly** define a symbol for the unknown answer and use it in later parts. Note: this is an act of desperation – we often give multiple parts to guide you through a problem.
- **Show your work!**

1. Rolling marbles [20 pts]. A uniform solid sphere of radius R and mass M is rolled up an incline set at an angle θ with respect to horizontal. The surface of the incline has a coefficient of friction μ. At $t = 0$, the marble's linear velocity is v_0 (parallel to the incline), and its angular velocity, ω, is zero. At what time does the marble start to roll without slipping? (Hint: work in coordinates parallel and perpendicular to the surface of the incline.)

2. Bullet and Hoop [30 pts]. A thin hoop of mass m and radius R is lying flat on a frictionless table. It is hit by a bullet, also of mass m, moving at speed v tangential to the hoop. The bullet lodges in the hoop as shown.

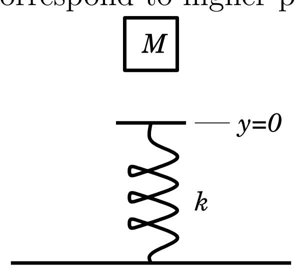


a) [10 pts] Where is the center of mass of the bullet+hoop system just after the collision? You may give your answer relative to the center of the hoop.

b) [10 pts] What is the moment of inertia I of the bullet+hoop system around its center of mass? Treat the bullet as a point particle of negligible size.

c) [10 pts] What is the rotational speed ω of the bullet+hoop system about the center of mass after the collision?

3. Mass on Spring [30 pts]. A stiff but light spring of spring constant k is embedded in the ground as shown. Its equilibrium length is such that the plate at the top of the spring is at height $y = 0$. Let positive y correspond to higher positions.

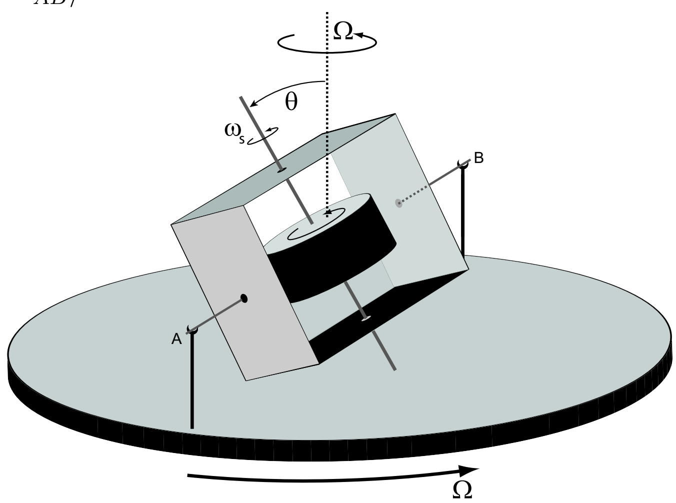


a) [10 pts] If a mass M is placed gently onto the spring, what is the offset in the spring's equilibrium position Δy relative to its initial equilibrium height $y = 0$? Give your answer in terms of the constants provided, and the acceleration due to gravity g.

b) [10 pts] If the mass M is dropped from a height $y = h$ above $y = 0$ onto the spring, what is the maximum compression y_{max} of the spring?

c) [10 pts] If the mass sticks to the spring when it lands on it, what are the final period and amplitude of the oscillations of the mass on the spring in the scenario described in part b)? (Note: since the spring and plate are light, the stickiness of the impact does not cost any energy – the answer to b) is still valid.)

4. Gryoscope on a Turntable [30 pts]. A rapidly spinning disk with angular frequency ω_s is held as shown in the picture below. The disk has an angular momentum $L_s = I_s\omega_s$. The apparatus that holds the disk may be considered massless and is mounted on a slowly rotating turntable with angular frequency $\Omega \ll \omega_s$. Note that the disk is free to rotate about the AB axis. This means that the *net* angular momentum along the AB direction must remain constant, or $d\,L_{AB}/dt = 0$.



a) [5 pts] Ignoring the rotation of the turntable, imagine that the spinning disk accelerates in θ. What is the component of the resulting torque along the AB axis in terms of I_s, θ, and its derivatives? You may treat the disk as planar. (Hint: it may help to recall the spinning coin problem done in lecture and, to find the appropriate I, the *form* of the moment of inertia tensor. A full calculation of $\overleftrightarrow{I}$ is by no means necessary.)

b) [5 pts] Now consider the rotation of the turntable as well. What is the component of torque along the AB axis that arises because L_s is now rotating at Ω?

c) [10 pts] The combined effects associated with parts (a) and (b) cause the disk to oscillate in θ. Considering the net torque along the AB axis, what is the differential equation that describes this oscillation?

d) [5 pts] For small θ, what is the oscillation frequency in terms of ω_s and Ω?

e) [5 pts] Imagine that there is a slight amount of friction in the pivots at A and B that damps the oscillation with time constant τ. For $t \gg \tau$, how are $\vec{\omega}_s$ and $\vec{\Omega}$ aligned?

5. Perturbed Orbits [30 pts]. A particle of mass m moves in a central force field where the force as a function of $\mathbf{r}$ is given by $\mathbf{F}(r) = -kr^n\hat{\mathbf{r}}$, where $n \geq 0$, and k is a constant with dimensions Nm^{-n}. The angular momentum of the particle about the origin is ℓ.

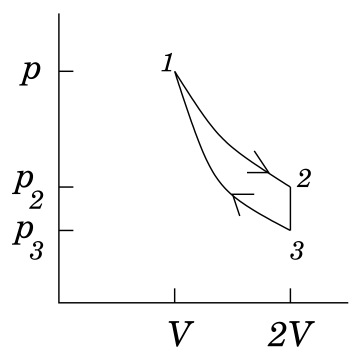
a) [5 pts] What is the potential energy $U(r)$ as a function of distance from the origin r? Take $U(0) = 0$.

b) [5 pts] What is the effective potential for the motion, $U_{\text{eff}}(r)$?

c) [10 pts] We now take the particle to be in a circular orbit with angular momentum ℓ as specified above. What is the period of one orbit T in terms of ℓ, n, m, and k? (Hint: the algebra can get a little messy and so please be careful!)

d) [10 pts] The particle is now given a small perturbation in the radial direction and begins oscillate in radius about the unperturbed circular orbit with period T_p. For what value of n does $T_p = T/3$? To solve this you will want to find T_p in terms of ℓ, n, m, and k.

6. Simple Engine [30 pts]. A heat engine uses a fixed amount of an ideal gas with $c_V = \frac{3}{2}R$ and $c_p = \frac{5}{2}R$. The cycle is shown in the figure. One of the curved legs is isothermal (constant temperature), the other is adiabatic (no heat transferred). The coordinates (V, p) without subscripts refer to point 1.



a) [10 pts] Which of the two curved legs, $1 \to 2$ or $3 \to 1$ is the adiabatic leg? To get credit you need to **briefly** explain your reasoning.

b) [10 pts] What is p_3 in terms of p (the pressure at point 1)?

c) [10 pts] What is the efficiency of this engine? (Hint: it helps to write all intermediate results like heats and works in terms of $(number) \times pV$.)

7. Nuclear Decay [30 pts]. A nucleus at rest in the lab and with mass M_0 decays into two identical fragments of rest mass $\frac{2}{5}M_0$ each. Assume that all energy goes into the fragments, neglect potential energy of any interaction between the fragments.

a) [10 pts] What is the **total** energy of each fragment?

b) [10 pts] What are the speeds of the fragments with respect to the lab frame?

c) [10 pts] What is the momentum of each fragment in terms of M_0 and c?

Physics 105 Formula Sheet

$$\mu = \frac{Mm}{M+m}, \qquad U_{\text{eff}} = \frac{l^2}{2\mu r^2} + U(r), \qquad \mathbf{F} = -\frac{GMm}{r^2}\,\hat{\mathbf{r}}, \qquad U = -\frac{GMm}{r}, \qquad T^2 = \frac{\pi^2 \mu A^3}{2C}$$

$$a_c = \frac{v^2}{r}, \qquad r = \frac{r_0}{1-\epsilon\cos\theta}, \qquad r_0 \equiv l^2/\mu C, \qquad \epsilon \equiv \sqrt{1+\frac{2El^2}{\mu C^2}}, \qquad C \equiv GMm$$

$$A = -C/E$$

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0 \quad \left(\text{or} = \frac{F}{m}\right)$$

$$\ddot{x} + \omega_0^2 x = 0, \qquad \omega_0^2 = k/m, \qquad x = A\cos(\omega_0 t + \phi), \qquad 2\pi f = \omega, \qquad T = 1/f$$

$$e^{i\phi} = \cos\phi + i\sin\phi, \qquad z = x + iy = re^{i\phi}, \qquad r^2 = zz^*, \qquad \tan\phi = \frac{\text{Im}z}{\text{Re}z}$$

$$y(x,t) = f(x \mp vt), \qquad y(x,t) = A\cos(kx \mp \omega t) \qquad v = \frac{\lambda}{T} = \frac{\omega}{k}, \qquad \lambda = \frac{2\pi}{k}$$

$$f'_{sound} = f_0(v_s \pm v_{rec})/(v_s \mp v_{src})$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}, \qquad \cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta, \qquad \cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\text{Lorentz transform}: \; x' = \gamma(x - vt), \quad y' = y, \quad z' = z, \quad t' = \gamma(t - xv/c^2)$$
$$x = \gamma(x' + vt'), \quad y = y', \quad z = z', \quad t = \gamma(t' + x'v/c^2)$$

$$\beta = v/c, \qquad \gamma = 1/\sqrt{1-\beta^2}, \qquad u_x = (u'_x + v)/(1 + vu'_x/c^2), \qquad u_y = u'_y/\gamma(1 + vu'_x/c^2)$$

$$E = \gamma m_0 c^2, \qquad p = \gamma m_0 v, \qquad E^2 = p^2c^2 + m_0^2 c^4, \qquad E = pc$$

For a planar object rotationally symmetric about z:

$$\overleftrightarrow{\mathrm{I}} = \begin{pmatrix} \int \rho y^2\, dV & 0 & 0 \\ 0 & \int \rho x^2\, dV & 0 \\ 0 & 0 & \int \rho(x^2+y^2)\, dV \end{pmatrix}$$