Physics 105 2$^{\text{nd}}$ Midterm Exam

December 7, 2005

Ninety minutes. Closed book. You may use a calculator.

Write your answers in an exam book. Be sure to put your name on the front of the book! Show your work, explain your reasoning, and $\boxed{\text{box}}$ your final answers. **You may use the equations on the front page without deriving them.**

There are three problems of varying lengths, with point values as indicated in the booklet. The total points value is 50.

Good luck.

After you have completed the exam, write and sign the honor pledge on the front of the exam book: **"I pledge my honor that I have not violated the Honor Code during this examination."**

When you are finished, be sure to hand your exam book directly to one of the instructors.

$$\mu = \frac{Mm}{M+m}, \qquad U_{\text{eff}} = \frac{l^2}{2\mu r^2} + U(r), \qquad \mathbf{F} = -\frac{GMm}{r^2}\,\hat{\mathbf{r}}, \qquad U = -\frac{GMm}{r}$$

$$a_c = \frac{v^2}{r}, \qquad r = \frac{r_0}{1-\epsilon\cos\theta}, \qquad r_0 \equiv l^2/\mu C, \qquad \epsilon \equiv \sqrt{1+\frac{2El^2}{\mu C^2}}, \qquad C \equiv GMm$$

$$E = -\frac{GMm}{A}, \qquad T^2 = \frac{\pi^2 A^3}{2(M+m)G}, \qquad 2\pi f = \omega = 2\pi/T, \qquad \ddot{x} + \omega_0^2 x = 0$$

$$\omega_0^2 = k/m, \qquad x = A\cos(\omega_0 t + \phi) = C\cos(\omega_0 t) + B\sin(\omega_0 t)$$

$$A = \frac{F_0}{m\sqrt{(\omega_0^2-\omega^2)^2 + (\omega\gamma)^2}}$$

$$\phi = \arctan\left(\frac{\omega\gamma}{\omega^2-\omega_0^2}\right)$$

$$f(x) = f(x_0) + (x-x_0)\frac{df}{dx}\Big|_{x_0} + \frac{1}{2}(x-x_0)^2\frac{df^2}{d^2x}\Big|_{x_0} + \ldots$$

$$e^{i\phi} = \cos\phi + i\sin\phi, \qquad z = x + iy = re^{i\phi}, \qquad r^2 = zz^*$$

$$\tan\phi = \frac{\mathrm{Im} z}{\mathrm{Re} z}, \qquad \ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0 \quad \left(\text{or} = \frac{F}{m}\right)$$

$$Q = \frac{E}{|\Delta E| \text{per rad}} = 2\pi \frac{E}{|\Delta E| \text{per period}} \approx \frac{\omega_0}{\gamma} \approx \frac{\omega_0}{\Delta w}$$

$$y(x,t) = f(x \mp vt), \qquad y(x,t) = A\sin(kx \mp \omega t)$$

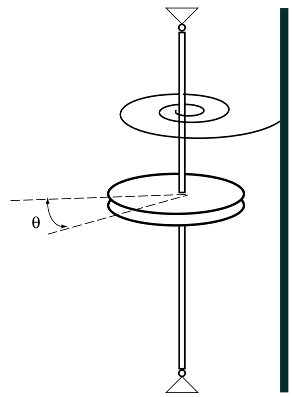
$$\frac{d^2y}{dt^2} - v^2\frac{d^2y}{dx^2} = 0, \qquad v = \frac{\lambda}{T} = \frac{\omega}{k}, \qquad \lambda = \frac{2\pi}{k}$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}, \qquad \cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta, \qquad \cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

Problem 1[15 points]

(K&K 6.19) A solid disk of mass M and radius R is affixed to a vertical shaft. The shaft is attached to a coil spring which exerts a restoring torque on it. This torque has magnitude $C\theta$, where θ is the angle measured from the static equilibrium position and C is a constant. In the following, you should neglect the mass of the shaft and spring and assume that there is no damping. Recall that the moment of inertia of a disk of radius R is $I = MR^2/2$.



a) (3 pts) What is the differential equation that describes the rotational motion of mass and what is the angular frequency (in rad/sec) of the oscillation, ω_a?

Suppose the disk is moving with $\theta(t) = \theta_0 \sin(\omega_a t)$ where ω_a is the angular frequency found above. At time $t = \pi/\omega_a$, a thin ring of sticky putty of the same mass M and radius R is dropped *concentrically* on the disk.

b) (2 pts) What is the new angular frequency of oscillation, ω_b?

c) (10 pts) What is the new amplitude of the oscillation?

Problem 2 [20 points]

A satellite of total mass m consists of two parts, A and B, with $m_A = m/4$ and $m_B = 3m/4$. It orbits the Earth (mass $M_E >> m$ and radius R_E) in a circular orbit of orbital radius $R \gg R_E$. In what follows, try to give your answers in terms of G, m, M_E, and R.

a) (3 pts) At times $t < 0$, when the whole satellite is in its circular orbit, what are its period T, total energy E, and angular momentum L?

At time $t = 0$, a massless spring causes the two parts of the satellite to fly apart. (The spring itself is **not** attached to either part.) The impulse from the spring is tuned exactly so that part A (mass $m/4$) is stopped completely dead ($\mathbf{v}_A = 0$ with respect to the Earth) and then falls radially towards the Earth.

b) (7 pts)At what time t_A does part A hit the Earth? Treat the Earth as a point mass with radius $R_E \ll R$. **Hint:** You can approximate the radial trajectory as a **very** elliptical orbit with the Earth at one focus.

c) (10 pts) What is the total energy E_B of part B (mass $3m/4$) at times $t > 0$ after the two parts have sprung apart? Compute the orbital semi-major axis a_B and eccentricity ϵ_B.

Problem 3. (15 points)

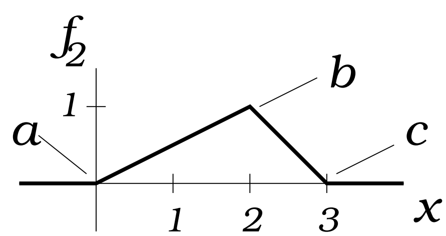
Consider a string running along the x-axis. At $t = 0$, the string has the shape $f_1(x) = (0.5\ m)\cos\left[(2\ m^{-1})x\right]$, with f_1 and x in meters. Consider a string running along the x axis.

a) (5 pts) The fixed shape f_1 travels in the positive x direction with speed $v = 3$ m/s. What is the function $y(x,t)$ that gives the vertical displacement of each part of the string as a function of time?

b) (5 pts) What are the speed and direction of motion of the *bit of string* at $x = 1$ m at $t = 2$ s?

Now consider a different wave shape pictured below. It is shown at $t = 0$; both f_2 and x are in meters. This shape moves to the **left** on a string at speed $v = 2$ m/s. The points a, b, and c represent features fixed to the *moving shape.*

$$f_2(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ x/2 & \text{for} \quad 0 \le x < 2\ m \\ 3\ m - x & \text{for} \quad 2\ m \le x < 3\ m \\ 0 & \text{for} \quad x \ge 3\ m \end{cases}$$



c) (5 pts) As a function of time, what is the velocity of a *bit of string* at $x = 1$ m. Start from time $t = -1$ s.