Physics 105 2$^{\text{nd}}$ Midterm Exam

December 6, 2006

Ninety minutes. Closed book. You may NOT use a calculator!

Write your answers in an exam book. Be sure to put your name on the front of the book! Show your work, explain your reasoning, and box your final answers. **You may use the equations on the front page without deriving them.**

There are three problems of varying lengths and difficulty, with point values as indicated in the booklet. The total points value is 50.

Good luck.

After you have completed the exam, write and sign the honor pledge on the front of the exam book: **"I pledge my honor that I have not violated the Honor Code during this examination."**

When you are finished, be sure to hand your exam book directly to one of the instructors.

$$\mu = \frac{Mm}{M+m}, \qquad U_{\text{eff}} = \frac{l^2}{2\mu r^2} + U(r), \qquad \mathbf{F} = -\frac{GMm}{r^2}\hat{\mathbf{r}}, \qquad U = -\frac{GMm}{r}$$

$$a_c = \frac{v^2}{r}, \qquad r = \frac{r_0}{1-\epsilon\cos\theta}, \qquad r_0 \equiv l^2/\mu C, \qquad \epsilon \equiv \sqrt{1+\frac{2El^2}{\mu C^2}}, \qquad C \equiv GMm$$

$$E = -\frac{GMm}{A}, \qquad T^2 = \frac{\pi^2 A^3}{2(M+m)G}, \qquad 2\pi f = \omega = 2\pi/T, \qquad \ddot{x} + \omega_0^2 x = 0$$

$$\omega_0^2 = k/m, \qquad x = A\cos(\omega_0 t + \phi) = C\cos(\omega_0 t) + B\sin(\omega_0 t)$$

$$A = \frac{F_0}{m\sqrt{(\omega_0^2 - \omega^2)^2 + (\omega\gamma)^2}}$$

$$\phi = \arctan\left(\frac{\omega\gamma}{\omega^2 - \omega_0^2}\right)$$

$$f(x) = f(x_0) + (x-x_0)\frac{df}{dx}\bigg|_{x_0} + \frac{1}{2}(x-x_0)^2\frac{df^2}{d^2x}\bigg|_{x_0} + \ldots$$

$$e^{i\phi} = \cos\phi + i\sin\phi, \qquad z = x + iy = re^{i\phi}, \qquad r^2 = zz^*$$

$$\tan\phi = \frac{\mathrm{Im}z}{\mathrm{Re}z}, \qquad \ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0 \quad \left(\mathrm{or} = \frac{F}{m}\right)$$

$$Q = \frac{E}{|\Delta E|\text{per rad}} = 2\pi\frac{E}{|\Delta E|\text{per period}} \approx \frac{\omega_0}{\gamma} \approx \frac{\omega_0}{\Delta w}$$

$$y(x,t) = f(x \mp vt), \qquad y(x,t) = A\sin(kx \mp \omega t)$$

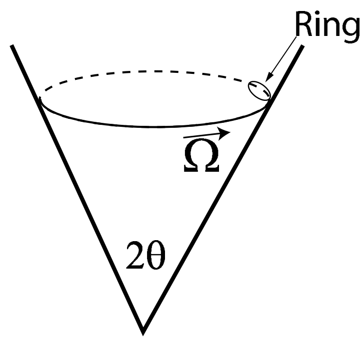
$$\frac{d^2y}{dt^2} - v^2\frac{d^2y}{dx^2} = 0, \qquad v = \frac{\lambda}{T} = \frac{\omega}{k}, \qquad \lambda = \frac{2\pi}{k}$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}, \qquad \cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta, \qquad \cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

Problem 1[20 points]

A ring of mass m and radius b rolls without slipping around the inside of an inverted cone of opening angle is 2θ as shown in the figure below. The radius of the ring's path is $R = h\tan(\theta)$ where h is the vertical height above the apex. The plane of the ring is always normal to the tangent plane of the cone at the point of contact. The forces acting on the ring are gravity, the normal force of the cone on the ring (N), and the frictional force (f) that keeps the ring from slipping to the apex of the cone. The angular frequency of rotation of the ring around the inside of the cone is Ω.



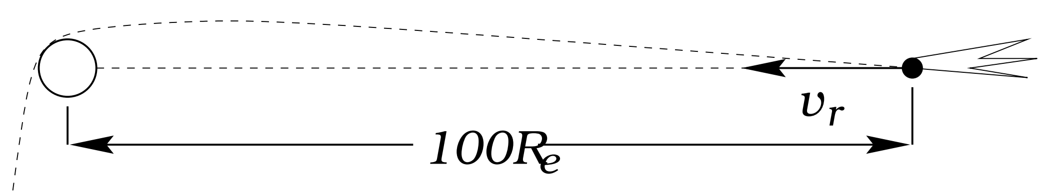
a) [5 pts] What is the torque (magnitude and direction) on the ring about its center of mass?

b) [7 pts] What is the frictional force, f, in terms of m, Ω, θ, h, & g?

c) [8 pts] What is Ω, in terms of θ, h, & g? You may assume that the dimensions of the ring are much smaller than the radius of the path the ring follows, $h\tan\theta$.

Problem 2.[15 points]

News Flash! Astronomers discover a medium-size comet ($m = 5 \times 10^{-12}$ earth mass) headed straight toward the center of the Earth. (In this problem **ignore** the Sun and the motion of the Earth.) Their observations indicate that the comet had essentially $v = 0$ with respect to the Earth when it was very far from the Earth. A band of endearing movie stars is enlisted to undertake a space mission to rendezvous with the comet when it is 100 Earth radii away and deflect the comet enough to miss the Earth.

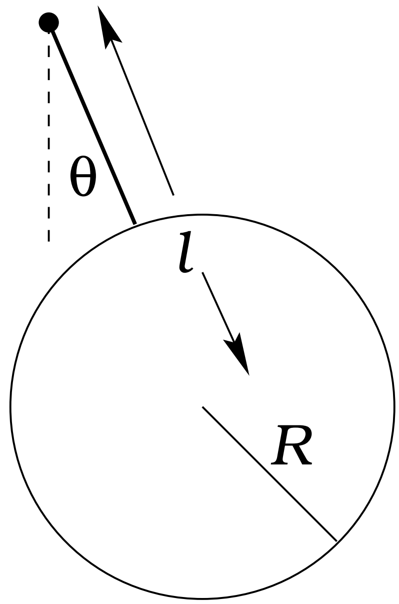


a) [5 pts] What is v_r, the speed of the comet when it reaches the rendezvous point?

b) [10 pts] What velocity component, $v_\perp$, perpendicular to its initial direction (and **in addition** to $\mathbf{v}_r$) must the comet be given so that it will just miss the Earth? (Neglect the thickness of the atmosphere.)

Problem 3.[15 points]

A physical pendulum is made from a uniform solid **sphere** of mass m and radius R attached to a thin, massless rod, of length $l - R$ as shown in the picture. The moment of inertia of the sphere about its center is $I_{cm} = \frac{2}{5}MR^2$.



a) [6 pts] What is ω_0, the angular frequency of small oscillations of the pendulum? There is no damping in this part of the problem.

Now, the sphere is unscrewed from the rod and dropped into a deep pool of water. It is noted that, as it falls through the water, it reaches a constant "terminal" speed v_t. The pendulum is then reassembled and the whole thing is immersed in the pool of water. (**Note:** assume that the sphere is dense enough that any buoyant forces are negligible.) We can now write the equation of motion for the angle the rod makes with the vertical as

$$\ddot{\theta} + \gamma\dot{\theta} + \omega_0^2\theta = 0$$

b) [4 pts] What is the relation between γ and v_t? (Hint: check your units!)

c) [5 pts] The submerged pendulum is held off to one side and released from rest from a small angle. It undergoes a number of oscillations as the amplitude decays. What is the new oscillation frequency ω_1? To get credit, you need to **derive** the new frequency from the above equation of motion.