Physics 105 2$^{\text{nd}}$ Midterm Exam

December 5, 2007

Ninety minutes. Closed book. You may NOT use a calculator!

Write your answers in an exam book. Be sure to put your name on the front of the book! Show your work, explain your reasoning, and $\boxed{\text{box}}$ your final answers. **You may use the equations on the front page without deriving them unless otherwise directed.**

There are three problems of varying lengths and difficulty, with point values as indicated in the booklet. The total points value is 50.

Good luck.

After you have completed the exam, write and sign the honor pledge on the front of the exam book: **"I pledge my honor that I have not violated the Honor Code during this examination."**

When you are finished, be sure to hand your exam book directly to one of the instructors.

$$\mu = \frac{Mm}{M+m}, \qquad U_{\text{eff}} = \frac{l^2}{2\mu r^2} + U(r), \qquad \mathbf{F} = -\frac{GMm}{r^2}\hat{\mathbf{r}}, \qquad U = -\frac{GMm}{r}$$

$$a_c = \frac{v^2}{r}, \qquad r = \frac{r_0}{1-\epsilon\cos\theta}, \qquad r_0 \equiv l^2/\mu C, \qquad \epsilon \equiv \sqrt{1+\frac{2El^2}{\mu C^2}}, \qquad C \equiv GMm$$

$$E = -\frac{GMm}{A}, \qquad T^2 = \frac{\pi^2 A^3}{2(M+m)G}, \qquad 2\pi f = \omega = 2\pi/T, \qquad \ddot{x} + \omega_0^2 x = 0$$

$$\omega_0^2 = k/m, \qquad x = A\cos(\omega_0 t + \phi) = C\cos(\omega_0 t) + B\sin(\omega_0 t)$$

$$A = \frac{F_0}{m\sqrt{(\omega_0^2-\omega^2)^2 + (\omega\gamma)^2}}$$

$$\phi = \arctan\left(\frac{\omega\gamma}{\omega^2 - \omega_0^2}\right)$$

$$f(x) = f(x_0) + (x-x_0)\frac{df}{dx}\Big|_{x_0} + \frac{1}{2}(x-x_0)^2\frac{df^2}{d^2x}\Big|_{x_0} + ...$$

$$e^{i\phi} = \cos\phi + i\sin\phi, \qquad z = x + iy = re^{i\phi}, \qquad r^2 = zz^*$$

$$\tan\phi = \frac{\mathrm{Im}z}{\mathrm{Re}z}, \qquad \ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0 \quad \left(\mathrm{or} = \frac{F}{m}\right)$$

$$Q = \frac{E}{|\Delta E|\text{per rad}} = 2\pi\frac{E}{|\Delta E|\text{per period}} \approx \frac{\omega_0}{\gamma} \approx \frac{\omega_0}{\Delta w}$$

$$y(x,t) = f(x \mp vt), \qquad y(x,t) = A\sin(kx \mp \omega t)$$

$$\frac{d^2y}{dt^2} - v^2\frac{d^2y}{dx^2} = 0, \qquad v = \frac{\lambda}{T} = \frac{\omega}{k}, \qquad \lambda = \frac{2\pi}{k}$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}, \qquad \cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta, \qquad \cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

Problem 1[15 points]

A hoop of mass M rolls in a straight line in the $+\hat{y}$ direction with velocity v. The $+\hat{z}$ direction is normal to the ground. By hitting it with a stick, its top is given a small impulse J_x in the $+\hat{x}$ direction. Assuming that the hoop is rolling fast enough to act like a gyroscope, what is the deflection angle of its path? Be sure to specify the magnitude and direction. You may neglect friction with the ground.

Problem 2.[15 points]

The starship *Enterprise* is in a circular orbit of radius R above the equator of a star of mass M. Suddenly, the star explodes, blasting away 1/4 of its mass in two symmetric jets from its poles thus missing the Enterprise and leaving its velocity and position unchanged. The remnant star is stationary and has mass 3M/4. We will assume that the blast occurs very quickly, blasting 1/4 of the star's mass to infinity instantaneously.

a) What is the shape of the *Enterprise's* orbit after the explosion? To get credit you must explain your answer.

b) What are the maximum and minimum distances from the remnant's center for the new orbit?

Problem 3.[20 points]

You are asked by GM to design the suspension for the new Corvette. You decide to use stiff European shocks and try critical damping. Here is the formula that you find best describes the vertical position y of the passanger compartment as the (poor) test driver races over a test track with a sinusoidal surface of amplitude A_0: (Take this equation as the starting point of your solution)

$$\ddot{y} + 2\omega_0\dot{y} + \omega_0^2 y = A_0\omega_0^2\cos(\omega t).$$

(ω_0 is the angular frequency of oscillation of the car before you had the shocks installed, ω is the angular frequency with which the wheels oscillate due to the sinusoidal bumps in the road.)

a) Derive formulas for the amplitude A and phase ϕ of the resulting oscillation

$$y = A\cos(\omega t + \phi)$$

of the passenger compartment as a function of ω, A_0 and ω_0. (Do not just use the final results for A and ϕ that we derived in class.)

b) Sketch A and ϕ as a function of ω. (A rough sketch is OK; make sure you show A_0 and ω_0 in your graph and the value of A, ϕ at $\omega = 0$, $\omega = \omega_0$, and ω large. Clearly indicate where A is maximal.)

c) For comparison sketch A as a function of ω for a lightly damped (old fashioned) Oldsmobile, indicating the behavior of A and ϕ at $\omega = 0$, $\omega = \omega_0$, and ω large.

d) Say, $f_0 = \omega_0/2\pi = 1$Hz and the bumps are 1m apart and 4cm high ($A_0 = 2$cm). At what horizontal speed (in miles per hour) is the vertical speed of the car in phase with the bumps i.e., $v_y \propto \cos(\omega t)$? What is the amplitude A at that speed? (1mi = 1.6km, 1h = 3600s)